Moriah College

2022

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using blue or black pen
- NESA-approved calculators may be used
- A reference sheet is provided
- For questions in Section II show relevant mathematical reasoning and/or calculations

Total Marks – **70**

Section I Questions 1 – 10 **10 marks**
Allow about 15 minutes for this section

Section II Questions 11 – 14 **60 marks**
Allow about 1 hour and 45 minutes for this section

Section 1

10 marks

Attempt Questions 1 – 10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

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1 Which of the following polynomials has a factor of $(x+1)$?

(A) $P(x)=x^3-5x^2+4$

(B) $P(x)=x^3-5x^2+6$

(C) $P(x)=x^3-x^2$

(D) $P(x)=x^4+x^2$

2 Which of the following is equivalent to $\dfrac{2\tan\theta}{1+\tan^2\theta}$?

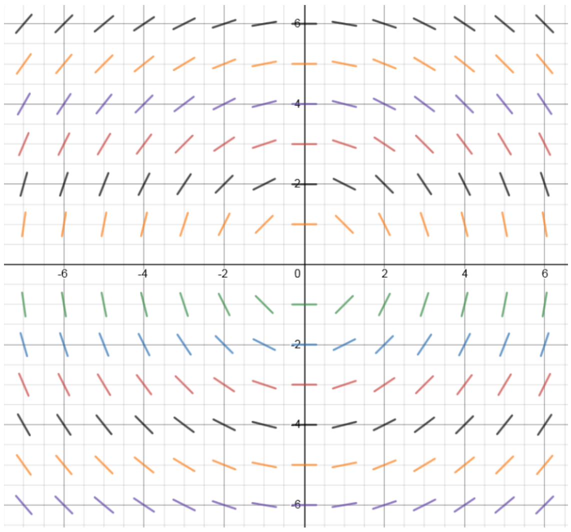
(A) $\cos 2\theta$

(B) $\sin 2\theta$

(C) $\tan 2\theta$

(D) $\cot 2\theta$

3 The slope field for the derivative of $y = f(x)$ is shown below.



Which of the following functions would have the same slope field?

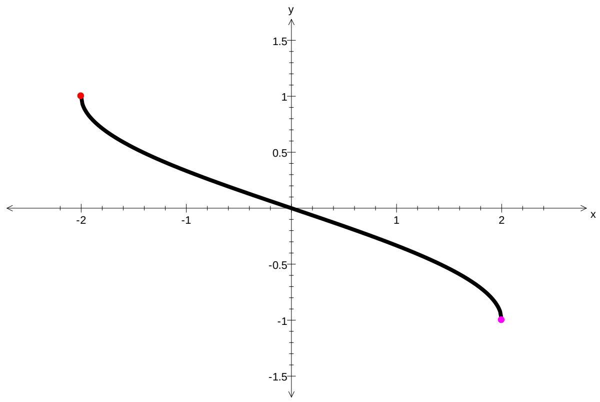
(A) $g(x) = 2f(x)$

(B) $g(x) = f(x+2)$

(C) $g(x) = 2 - f(x)$

(D) $g(x) = 2 + f(x)$

4 The function shown in the diagram below has equation $y = A\sin^{-1} Bx$.
Which of the following is true?



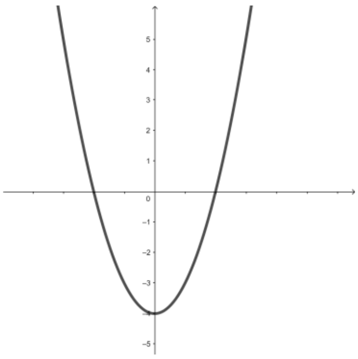
(A) $A = 1,\ B = \dfrac{1}{2}$

(B) $A = -1,\ B = 2$

(C) $A = \dfrac{-2}{\pi},\ B = \dfrac{1}{2}$

(D) $A = \dfrac{2}{\pi},\ B = 2$

5



The graph of $y = f\left(x\right)$ is shown above

How many real solutions does the equation $\left[f\left(x\right)\right]^2 = x$ have?

(A) 1

(B) 2

(C) 3

(D) 4

6 The definite integral $\int_{e}^{e^2} \frac{2}{x(\log_e x)^2} dx$ is evaluated using the substitution $u = \log_e x$.

Which of the following gives the correct value of the integral?

(A) $2\left(\frac{1}{e} - \frac{1}{e^2}\right)$

(B) $2\left(\frac{1}{e^2} - \frac{1}{e}\right)$

(C) -1

(D) 1

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7 A particle is moving in a straight line such that its displacement (x metres) from the origin after t seconds is given by $x = \sin^2 t$.

which of the following best describes the motion of the particle when $t = \frac{2\pi}{3}$?

(A) The particle is moving to the right with increasing speed.

(B) The particle is moving to the right with decreasing speed.

(C) The particle is moving to the left with increasing speed.

(D) The particle is moving to the left with decreasing speed.

8 The derivative of $y = \tan^{-1}\left[2f\left(x\right)\right]$ is?

(A) $\dfrac{dy}{dx} = \dfrac{1}{1+\left[f\left(x\right)\right]^2}$

(B) $\dfrac{dy}{dx} = \dfrac{2}{1+4\left[f\left(x\right)\right]^2}$

(C) $\dfrac{dy}{dx} = \dfrac{f'\left(x\right)}{1+4\left[f\left(x\right)\right]^2}$

(D) $\dfrac{dy}{dx} = \dfrac{2f'\left(x\right)}{1+4\left[f\left(x\right)\right]^2}$

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9 A particle moves in a straight line. Its displacement from the origin at any time t is given by $x = 3\cos 2t + 4\sin 2t$.

The maximum velocity attained by the particle is

(A) 10

(B) 7

(C) 5

(D) 4

10 A bag contains ten tickets numbered 1, 2, 3……,10.

Joanna chooses a ticket at random, records the number and **<u>replaces</u>** the ticket in the bag. The process is repeated two more times so that a total of three tickets are drawn.

What is the probability that the number on the first ticket drawn is higher than the second AND the second is higher than the third?

(A) $\frac{3}{25}$

(B) $\frac{1}{6}$

(C) $\frac{18}{25}$

(D) $\frac{4}{5}$

Section II

60 marks

Attempt Questions 11 – 14

Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Consider the polynomial $P(x) = x^3 + 6x^2 + 5x - 12$

(i) Show that $(x-1)$ is a factor of $P(x)$ **1**

(ii) Hence, find all roots of the equation $P(x) = 0$ **3**

(b) The vectors $\underset{\sim}{u} = \begin{pmatrix} 6 \\ a \end{pmatrix}$ and $\underset{\sim}{v} = \begin{pmatrix} a+1 \\ -9 \end{pmatrix}$ are perpendicular. **2**

Find the value of *a*.

(c) A school has ten prefects. John and Alex are two of those prefects.

A committee of six prefects is to be formed to organise a function.

(i) How many different committees can be formed? **1**

(ii) What is the probability that the committee formed contains John but does not contain Alex? **2**

Question 11 continues on the next page

Question 11 (continued)

(d) You are given that $\sin\theta = \dfrac{4}{5}$ where $90° < \theta < 180°$

(i) Find the exact value of $\tan\theta$ **1**

(ii) Find the exact value of $\tan 2\theta$ **2**

(e) Solve the inequality $\dfrac{x+3}{x^2-1} \leq 0$ **3**

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Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) Find $\int 2\cos^2 \frac{x}{2}\, dx$ **2**

(b) Find the equation of the curve $y = f\left(x\right)$ that passes through the point **3**

$(0, -\frac{\pi}{2})$ with gradient function $\frac{dy}{dx} = \frac{3}{\sqrt{16-9x^2}}$.

(c)

(i) Use mathematical induction to prove that for $n \geq 2$, **3**

$$(1-\frac{1}{2^2})\times(1-\frac{1}{3^2})\times(1-\frac{1}{4^2})\times......\times(1-\frac{1}{n^2}) = \frac{n+1}{2n}$$

(ii) Hence, evaluate $\frac{3}{4}\times\frac{8}{9}\times\frac{15}{16}\times......\times\frac{9999}{10000}$ **1**

(d) The rate of change of the share price \$$P$ of a company over a twelve month period can be modelled by the differential equation $\frac{dP}{dt} = k(P-8)$, where k is a constant and t is the time in months.

The share price was \$20 at the start of the period.

(i) **By solving the differential equation** show that $P = 8 + 12e^{kt}$. **3**

(ii) The share price increased to \$35 after 3 months. **1**

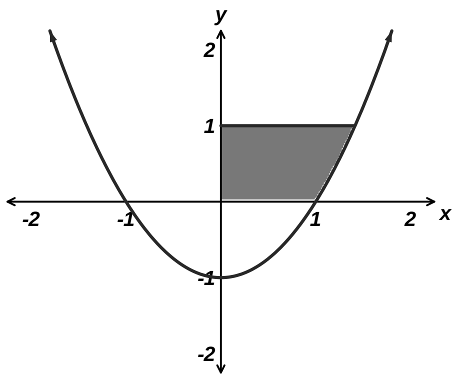
Show that $k = \frac{1}{3}\log_e\left(\frac{9}{4}\right)$

(iii) After T months the share price is increasing at a rate that is three times the initial rate. **2**

Find T correct to one decimal place.

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) **3**



The diagram above shows the shaded region bounded by $y = x^2 - 1$, the coordinate axes and the line $y = 1$. The region is rotated about the ***y*-axis** to form a solid.

Find the exact volume of the solid.

(b) Solve $\sqrt{2}\cos\left(x+\frac{\pi}{4}\right)+\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)=1$ for $-\pi \le x \le \pi$ **3**

(c) Let $\overrightarrow{OA}=\begin{pmatrix}6\\8\end{pmatrix}$ and $\overrightarrow{OB}=\begin{pmatrix}-3\\4\end{pmatrix}$ where O is the origin.

(i) Find the vector $\overrightarrow{AB}$ **1**

(ii) Find the angle between the two vectors **2**

(ii) Find $\left|proj_{\overrightarrow{OB}}\overrightarrow{OA}\right|$ **2**

(d) Consider the cubic polynomial $f(x) = ax^3 + bx^2 + cx + d$ where a, b, c and d are real numbers and $a \neq 0$. Let α, β and γ be zeros of $f(x)$.

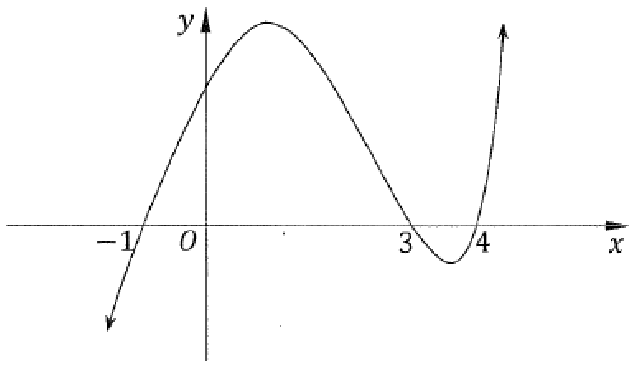
(i) Write down an expression for $\alpha + \beta + \gamma$. **1**

All cubic polynomial functions have a single point of inflexion when the second derivative is equal to zero.

(ii) Using part (i), or otherwise, show that the x-coordinate of the point of inflexion on the curve $y = f(x)$ is given by **2**

$$x = \frac{\alpha + \beta + \gamma}{3}.$$

(iii) The cubic polynomial below has x-intercepts at -1, 3 and 4. Find the x-coordinate of the point of inflexion of the cubic polynomial. **1**



Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) A circular metal plate is heated so that its radius is increasing at a constant rate of 0.005 m/s. **3**

At what rate is the area of the circular surface of the plate increasing when its radius is 2 metres? (give answer in m^2 / s correct to 2 dp)

(b) A particle is projected with a velocity of $40\,ms^{-1}$ at an angle of 30° above the horizontal.

Its position t seconds after projection is given by the displacement vector

$$\underset{\sim}{r}(t) = \left(20\sqrt{3}\,t\right)\underset{\sim}{i} + \left(20t - 5t^2\right)\underset{\sim}{j}$$

(i) Find the maximum height reached by the particle. **2**

(ii) At time $t = 1$ seconds, the particle is travelling on a path inclined at an angle α to the horizontal. **3**

Find α correct to the nearest degree and the exact speed of the particle at this time.

(iii) A second particle is projected with a velocity V at an angle of 60° **2**

This particle reaches the same maximum height as the first particle.

Find the value of V, assuming again that $g = 10\,m / s^2$.

(c) Consider the function $f\left(x\right)=\log_4 x+\log_5 x \;\; (x>0)$.

(i) Explain why the function has an inverse function. **1**

(ii) Show that the inverse function is given by **3**

$$f^{-1}(x)=e^{x\left(\frac{\ln 4\times\ln 5}{\ln 20}\right)}.$$

(iii) Hence, or otherwise, solve $\log_4 x+\log_5 x=2$. **1**

Write your answer correct to three decimal places.

End of Examination

SUGGESTED SOLUTIONS

Moriah 2022 Mathematics Extension 1 Trial HSC Examination

Section 1

10 marks

Questions 1 – 10 (1 mark each)

Question 1 (1 mark)

Solution	**Answer**	**Mark**
$P(-1)=\left(-1\right)^3-5\left(-1\right)^2+6=0$	**B**	**1**

Question 2 (1 mark)

Solution	**Answer**	**Mark**
Let $t=\tan\theta$ then $\dfrac{2\tan\theta}{1+\tan^2\theta}=\dfrac{2t}{1+t^2}$ $=\sin 2\theta$	**B**	**1**

Question 3 (0 marks)

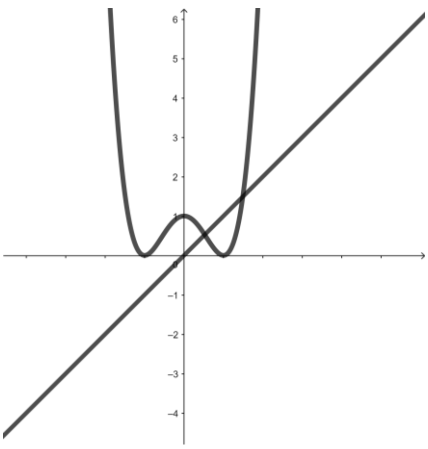
Solution	**Answer**	**Mark**
	No solutions	**0**

Question 4 (1 mark)

Solution	Answer	Mark
$A=\frac{-2}{\pi}$ Domain is $D:-1\le Bx\le 1$ $-\frac{1}{B}\le x\le\frac{1}{B}$ $\therefore\frac{1}{B}=2$ $B=\frac{1}{2}$	**C**	**1**

Question 5 (1 mark)

Solution	Answer	Mark
From graph below 2 real solutions	**B**	**1**



Question 6 (1 mark)

Solution	**Answer**	**Mark**
$\int_e^{e^2} \frac{2}{x(\log_e x)^2} dx = 2\int_e^{e^2} \frac{1}{(\log_e x)^2} \times \frac{1}{x} dx$ $= 2\int_1^2 \frac{1}{u^2} du$ $= 2\int_1^2 u^{-2} du$ $= 2\left[\frac{u^{-1}}{-1}\right]_1^2$ $= -2\left[\frac{1}{u}\right]_1^2$ $= -2\left(\frac{1}{2} - 1\right)$ $= 1$	**D**	**1**

Question 7 (1 mark)

Solution	**Answer**	**Mark**
$x = \sin^2 t$ $\dot{x} = \frac{dx}{dt} = 2\sin t\cos t = \sin 2t$ $\ddot{x} = 2\cos 2t$ when $t = \frac{2\pi}{3}$ $\dot{x} = \frac{dx}{dt} = \sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2} \Rightarrow$ particle moving to left $\ddot{x} = 2\cos\left(\frac{4\pi}{3}\right) = -1 \Rightarrow$ acceleration directed left	**C**	**1**

Question 8 (1 mark)

Solution	Answer	Mark
If $y=\tan^{-1}\left[2f(x)\right]$ $\frac{dy}{dx}=\frac{1}{1+\left[2f(x)\right]^2}\times 2f'(x)$ $=\frac{2f'(x)}{1+4\left[f(x)\right]^2}$	**D**	**1**

Question 9 (1 mark)

Solution	Answer	Mark
$v=\dot{x}$ $v=-6\sin 2t+8\cos 2t$ Maximum v occurs at the maximum amplitude of $R\cos(2t+\propto)$ where $R=\sqrt{a^2+b^2}$ $R=\sqrt{(-6)^2+8^2}$ $=\sqrt{100}$ $=10$	**C**	**1**

Question 10 (1 mark)

Solution	Answer	Mark
Total number of outcomes $=10\times10\times10=1000$ Number of ways of picking 3 distinct numbers $=10\times9\times8$ Number of arrangements of each set of 3 distinct numbers $=6$ $\therefore\frac{1}{6}$ of arrangements will be in descending order $\therefore$ Probability $=\frac{1}{6}\times\frac{10\times9\times8}{1000}=\frac{3}{25}$	**A**	**1**

Question 11 (15 marks)

(a) (i) (1 mark)

Criteria	Marks
• Provides correct answer.	**1**

Sample answer:

$$\begin{aligned} P(1) &= 1^3 + 6 \times 1^2 + 5 \times 1 - 12 \\ &= 1 + 6 + 5 - 12 \\ &= 0 \end{aligned}$$

(a) (ii) (3 mark)

Criteria	Marks
• Provides correct answer. • Makes progress solving the quadratic factor equal to 0. • Correctly factorises the expression using the $x - 1$ factor.	**3** **2** **1**

Sample answer:

$$\begin{aligned} P(x) &= (x-1)(x^2 + 7x + 12) \\ 0 &= (x-1)(x+3)(x+4) \end{aligned}$$

$$x = 1,\ x = -3,\ x = -4$$

(b) (2 marks)

Criteria	Marks
• Provides correct answer.	**2**
• Recognises that dot product is zero.	**1**

Sample answer:

$$Perpendicular \Rightarrow \begin{pmatrix} 6 \\ a \end{pmatrix} \bullet \begin{pmatrix} a+1 \\ -9 \end{pmatrix} = 0$$

$$6a + 6 - 9a = 0$$

$$a = 2$$

(c) (i) (1 mark)

Criteria	Marks
• Correct answer.	**1**

Sample answer:

Number of different committees $= {}^{10}C_6 = 210$

(c)(ii) (2 marks)

Criteria	Marks
• Provides correct answer.	**2**
• Makes some progress in finding the number of committees containing Jun but not Alex.	**1**

Sample answer:

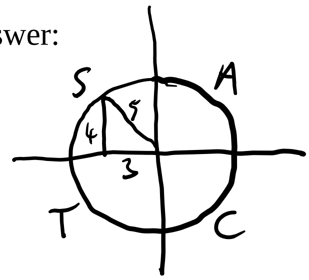
Jun is chosen. 5 other prefects chosen in ${}^{8}C_5$ways (given Alex not chosen)

$$\text{Prob} = \frac{{}^{8}C_5}{{}^{10}C_6} = \frac{4}{15}$$

(d) (i) (1 mark)

Criteria	Marks
• Provides correct answer.	**1**

Sample answer:



$$\tan\theta = -\frac{4}{3}$$

(d) (ii) (2 marks)

Criteria	Marks
• Provides correct answer. • Attempts to use t-formula.	**2** **1**

Sample answer:

$$\begin{aligned}
\tan 2\theta &= \frac{2\tan\theta}{1-\tan^2\theta} \\
&= \frac{2\times -\frac{4}{3}}{1-\left(-\frac{4}{3}\right)^2} \\
&= \frac{-\frac{8}{3}}{1-\frac{16}{9}} \\
&= \frac{-24}{9-16} \\
&= \frac{-24}{-7} \\
&= \frac{24}{7}
\end{aligned}$$

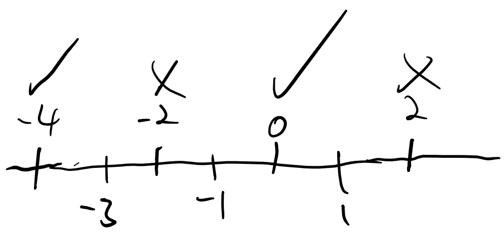
(e) (3 marks)

Criteria	Marks
• Correctly solves the inequation.	**3**
• Finds the solution that is equal to 0.	**2**
• Correctly identifies the values of x that are not valid.	**1**

Sample answer:

$$\begin{aligned}
x^2 - 1 &\neq 0 \\
x^2 &\neq 1 \\
x &\neq \pm 1
\end{aligned}$$

$$\begin{aligned}
x + 3 &= 0 \\
x &= -3
\end{aligned}$$



$x \leq -3,\ -1 < x < 1$

Question 12 (15 marks)

(a) (2 marks)

Criteria	Marks
• Correct answer (no marks lost for no $+c$)	**2**
• Correct use of $\cos 2\theta$ result	**1**

Sample answer:

$$\begin{aligned}\cos x &= 2\cos^2\frac{x}{2} - 1\\ 2\cos^2\frac{x}{2} &= 1 + \cos x\\ \int 2\cos^2\frac{x}{2}\,dx &= \int (1 + cos\ x)\,dx\\ &= x + \sin x + c\end{aligned}$$

(b) (3 marks)

Criteria	Marks
• Provides correct solution	**3**
• Substitutes correctly to find the constant of integration.	**2**
• Integrates the gradient function correctly.	**1**

Sample answer:

$$\begin{aligned}\frac{dy}{dx} &= \frac{3}{\sqrt{16 - 9x^2}}\\ &= \frac{3}{\sqrt{4^2 - (3x)^2}}\\ &= \frac{3}{4\sqrt{1 - \left(\frac{3x}{4}\right)^2}}\\ y &= \sin^{-1}\left(\frac{3x}{4}\right) + C\end{aligned}$$

At $\left(0, -\frac{\pi}{2}\right)$:

$$\begin{aligned}-\frac{\pi}{2} &= \sin^{-1}(0) + C\\ -\frac{\pi}{2} &= 0 + C\\ C &= -\frac{\pi}{2}\end{aligned}$$

$$\therefore y = \sin^{-1}\left(\frac{3x}{4}\right) - \frac{\pi}{2}$$

(c) (i) (3 marks)

Criteria	Marks
• Correctly proves the result.	**3**
• Uses the correct assumption step in attempting to prove the statement is true for $n = k + 1$	**2**
• Correctly proves that the statement is true when $n = 2$.	**1**

Sample answer:

Step1: Prove true for $n = 1$

$$LHS = 1 - \frac{1}{2^2}$$
$$= 1 - \frac{1}{4}$$
$$= \frac{3}{4}$$

$$RHS = \frac{2+1}{2 \times 2}$$
$$= \frac{3}{4}$$

$\therefore$ true for $n = 2$

Step2: Assume true for $n = k$

$$(1 - \frac{1}{2^2}) \times (1 - \frac{1}{3^2}) \times (1 - \frac{1}{4^2}) \times \ldots\ldots \times (1 - \frac{1}{k^2}) = \frac{k+1}{2k}$$

Step3: Prove true for $n = k + 1$

when $n = k + 1$

$$LHS = \left(1 - \frac{1}{2^2}\right) \times \left(1 - \frac{1}{3^2}\right) \times \left(1 - \frac{1}{4^2}\right) \times \ldots\ldots \times \left(1 - \frac{1}{k^2}\right) \times \left(1 - \frac{1}{(k+1)^2}\right)$$
$$= \frac{k+1}{2k} \times \left(1 - \frac{1}{(k+1)^2}\right) \text{ from Step 2}$$
$$= \frac{k+1}{2k} - \frac{1}{2k(k+1)}$$
$$= \frac{(k+1)^2 - 1}{2k(k+1)} = \frac{k^2 + 2k}{2k(k+1)}$$
$$= \frac{k(k+2)}{2k(k+1)}$$
$$= \frac{k+2}{2k+2}$$

$$RHS = \frac{k+1+1}{2(k+1)}$$
$$= \frac{k+2}{2k+2}$$

$\therefore$ true by mathematical induction

(c) (ii) (1 mark)

Criteria	Marks
• Provides correct answer.	**1**

Sample answer:

$$n^2 = 10000$$
$$n = 100$$

$$\frac{3}{4} \times \frac{8}{9} \times \frac{15}{16} \times \dots \dots \times \frac{9999}{10000} = \frac{100+1}{2 \times 100}$$
$$= \frac{101}{200}$$

(d) (i) (3 marks)

Criteria	Marks
• Correctly shows the result.	**3**
• Correct integration	**2**
• Separates the differential equation and makes some attempt to integrate.	**1**

Sample answer:

$$\frac{dP}{dt} = k(P-8)$$
$$\frac{dP}{P-8} = kdt$$
$$\int \frac{1}{P-8} dP = \int kdt$$
$$\log_e (P-8) = kt + c$$
$$t = 0, P = 20 \Rightarrow c = \log_e 12$$
$$\log_e (P-8) - \log_e 12 = kt$$
$$\log_e \left(\frac{P-8}{12} \right) = kt$$
$$\frac{P-8}{12} = e^{kt}$$
$$P = 8 + 12e^{kt}$$

(d) (ii) (1 mark1)

Criteria	Marks
• Provides correct answer.	**3**

Sample answer:

$$P = 8 + 12e^{kt}$$

when $t = 3, P = 35$

$$\begin{aligned} 35 &= 8 + 12e^{3k} \\ e^{3k} &= \frac{27}{12} = \frac{9}{4} \\ k &= \frac{1}{3} log_e \frac{9}{4} \end{aligned}$$

(d) (iii) (2 marks)

Criteria	Marks
• Provides correct answer.	**2**
• Finds initial rate of change and makes some further progress.	**1**

Sample answer:

$$\begin{aligned} \text{rate} &= \frac{dP}{dt} = 12ke^{kt} \\ \text{Initial rate of change } &= 12k \\ 3 \times \text{ Initial rate of change } &= 36k \\ 12ke^{kt} &= 36k \\ e^{kt} &= 3 \\ e^{\left(\frac{1}{3} log_e \frac{9}{4}\right)T} &= 3 \\ \left(\frac{1}{3} log_e \frac{9}{4}\right)T &= log_e 3 \\ T &= 4.0642.. = 4.1 (\text{to one decimal place}) \end{aligned}$$

Question 13 (15 marks)

(a) (3 marks)

Outcomes Assessed: ME12-4

Targeted Performance Band: E2-E3

Criteria	Marks
• Provides correct answer.	**3**
• Provides the correct substitution into the integrated expression.	**2**
• Integrates the function correctly.	**1**

Sample answer:

$$V = \pi \int_0^1 x^2 dy$$
$$= \pi \int_0^1 (y+1)\, dy$$
$$= \pi \left[\frac{y^2}{2} + y \right]_0^1$$
$$= \frac{3\pi}{2} \text{ units}^3$$

(b) (3 marks)

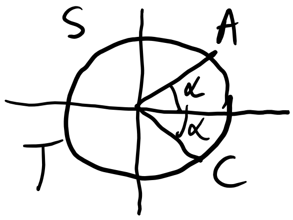
Criteria	Marks
• Provides correct answer.	**3**
• Simplifies the expression to find $\cos x = \frac{1}{2}$	**2**
• Attempts to expand the expression using the compound angle formula for cosine.	**1**

Sample answer:

$$\cos\left(x + \frac{\pi}{4}\right) + \cos\left(x - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$
$$\cos x \cos\frac{\pi}{4} - \sin x \sin\frac{\pi}{4} + \cos x \cos\frac{\pi}{4} + \sin x \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$
$$\frac{2}{\sqrt{2}}\cos x = \frac{1}{\sqrt{2}}$$
$$\cos x = \frac{1}{2}$$

Solve:

$$\cos\alpha = \frac{1}{2}$$
$$\alpha = \cos^{-1}\frac{1}{2}$$
$$\alpha = \frac{\pi}{3}$$



$$\therefore x = -\frac{\pi}{3}, \frac{\pi}{3}$$

(c) (i) (1 mark)

Criteria	Marks
• Provides correct answer.	**1**

Sample answer:

$$\vec{AB} = \begin{pmatrix} -3 \\ 4 \end{pmatrix} - \begin{pmatrix} 6 \\ 8 \end{pmatrix}$$
$$= \begin{pmatrix} -3-6 \\ 4-8 \end{pmatrix}$$
$$= \begin{pmatrix} -9 \\ -4 \end{pmatrix}$$

(c) (ii) (2 marks)

Criteria	Marks
• Provides correct answer.	**2**
• Finds dot product of vectors.	**1**

Sample answer:

$$\vec{OA} \bullet \vec{OB} = \binom{6}{8} \bullet \binom{-3}{4} = 6 \times -3 + 8 \times 4 = 14$$

$$\begin{aligned} \left|\vec{OA}\right| \times \left|\vec{OB}\right| cos\,\theta &= 14 \\ 10 \times 5 \cos\theta &= 14 \\ \cos\theta &= \frac{14}{50} \\ \theta &= \cos^{-1}\frac{7}{25} \\ &= 73°44' \end{aligned}$$

(c) (ii) (2 marks)

Criteria	Marks
• Provides correct answer.	**2**
• States the projection formula correctly.	**1**

Sample answer:

$$\begin{aligned} \left|proj_{\overrightarrow{OB}}\overrightarrow{OA}\right| &= \frac{\vec{OA} \bullet \vec{OB}}{\left|\vec{OB}\right|} \\ &= \frac{14}{5} \end{aligned}$$

(d) (i) (1 marks)

Criteria	Marks
• Correctly obtains result.	**1**

Sample answer:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

(d) (ii) (2 marks)

Criteria	Marks
• Correctly obtains result.	**2**
• Correctly finds the second differential.	**1**

Sample answer:

$$\begin{aligned} f'(x) &= 3ax^2 + 2bx + c \\ f''(x) &= 6ax + 2b \\ 0 &= 6ax + 2b \\ 6ax &= -2b \\ x &= -\frac{2b}{6a} \\ x &= -\frac{b}{3a} \\ x &= \frac{\alpha + \beta + \gamma}{3} \end{aligned}$$

(d) (iii) (1 marks)

Criteria	Marks
• Correctly obtains result.	**1**

Sample answer:

$$\begin{aligned} x &= \frac{\alpha + \beta + \gamma}{3} \\ &= \frac{-1 + 3 + 4}{3} \\ &= \frac{6}{3} \\ &= 2 \end{aligned}$$

Question 14 (15 marks)

(a) (3 marks)

Outcomes assessed: ME12-2

Targeted Performance Band: E3

Criteria	Marks
• Provides correct answer.	**3**
• Finds an expression for dA/dr and sets up the correct related rate.	**2**
• Makes some progress toward solving the problem.	**1**

Sample answer:

$$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$$
$$A = \pi r^2$$
$$\frac{dA}{dr} = 2\pi r$$

When $r = 2$:

$$\frac{dA}{dr} = 4\pi$$
$$\frac{dA}{dt} = 4\pi \times 0.005$$
$$= 0.06 \text{ to 2 d.p.}$$

(b) (i) (2 marks)

Outcomes assessed: ME12-2

Targeted Performance Band: E3

Criteria	Marks
• Provides correct answer.	**2**
• Finds vertical velocity.	**1**

Sample answer:

Maximum height when $\dot{y} = 0$

$20 - 10t = 0$

$t = 2$

$y_{\max} = 20(2) - 5(2)^2 = 20$ metres

(b) (ii) (3 marks)

Outcomes assessed: ME12-2

Targeted Performance Band: E4

Criteria	Marks
• Provides correct answers.	**3**
• Finds correct speed.	**2**
• Finds horizontal or vertical velocity and makes some further progress.	**1**

Sample answer:

When $t = 1$

$\dot{x} = 20\sqrt{3}$, $\dot{y} = 10$

$\tan\alpha = \dfrac{10}{20\sqrt{3}}$

$\alpha = 16^{\circ}$ $\left(\text{nearest degree}\right)$

speed $= \sqrt{\left(20\sqrt{3}\right)^2 + 10^2} = 10\sqrt{13}\, ms^{-1}$

(b) (iii) (2 marks)

Outcomes assessed: ME12-2

Targeted Performance Band: E4

Criteria	Marks
• Provides correct answers.	**2**
• Finds correct expression for t.	**1**

Sample answer:

$$\ddot{x} = 0, \ddot{y} = -10$$
$$\dot{x} = V\cos 60^{\circ}, \dot{y} = -10t + V\sin 60^{\circ}$$
$$\dot{x} = \frac{V}{2}, \dot{y} = -10t + \frac{V\sqrt{3}}{2}$$
$$x = \frac{Vt}{2}, y = -5t^2 + \frac{Vt\sqrt{3}}{2}$$

Particle reaches maximum height when $\dot{y} = 0$

$$0 = -10t + \frac{V\sqrt{3}}{2}$$
$$10t = \frac{V\sqrt{3}}{2}$$
$$t = \frac{V\sqrt{3}}{20}$$

When $t = \frac{V\sqrt{3}}{20}, y = 20$ so:

$$20 = -5\left(\frac{V\sqrt{3}}{20}\right)^2 + \frac{V\sqrt{3}}{2}\left(\frac{V\sqrt{3}}{20}\right)$$
$$20 = \frac{-15V^2}{400} + \frac{3V^2}{40}$$
$$20 = \frac{-15V^2}{400} + \frac{30V^2}{400}$$
$$15V^2 = 8000$$
$$V^2 = \frac{1600}{3}$$
$$V = \frac{40}{\sqrt{3}} m/s$$

(c) (i) (1 mark)

Criteria	Marks
• Provides correct reason.	**1**

Sample answer:

$y = \log_4 x$ is increasing for all $x > 0$

$y = \log_5 x$ is increasing for all $x > 0$

$\therefore\ f(x) = \log_4 x + \log_5 x$ is increasing for all $x > 0$

$\therefore\ f(x) = \log_4 x + \log_5 x$ is has an inverse function.

(c) (ii) (3 marks)

Criteria	Marks
• Correctly obtains result.	**3**
• Uses change of base formula to transform equation below or equivalent progress.	**2**
• Makes some progress towards result (e.g: writes $x = \log_4 y + \log_5 y$)	**1**

Sample answer:

$$f(x) = \log_4 x + \log_5 x$$
$$y = \log_4 x + \log_5 x$$
for inverse change x and y
$$x = \log_4 y + \log_5 y$$
$$= \frac{\ln y}{\ln 4} + \frac{\ln y}{\ln 5}$$
$$= \frac{(\ln 4 + \ln 5)\ln y}{\ln 4 \times \ln 5}$$
$$x = \frac{\ln 20 \times \ln y}{\ln 4 \times \ln 5}$$
$$\ln y = \frac{x(\ln 4 \times \ln 5)}{\ln 20}$$
$$y = e^{x\left(\frac{\ln 4 \times \ln 5}{\ln 20}\right)}$$
$$i.e\ f^{-1}(x) = e^{x\left(\frac{\ln 4 \times \ln 5}{\ln 20}\right)}$$

(c) (iii) (1 mark)

Criteria	Marks
• Correct answer (no marks deducted for not writing correct to three decimal places).	**1**

Sample answer:

$$x = f^{-1}(2)$$
$$= e^{2\left(\frac{\ln 4 \times \ln 5}{\ln 20}\right)}$$
$$= 4.43512.....$$
$$= 4.435 \text{ (to 3 decimal places)}$$